B.SC. FOURTH SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics | **Course ID: 42112**

Course Code: SH/MTH/402/C-9 | **Course Title: Multivariate Calculus**

Full Marks: 40 | **Time: 2 Hours**

<u>The figures in the margin indicate full marks</u>

Symbols and Notations have their usual meaning

1. Answer *any five* of the following questions: **2 × 5=10**

(a) Examine the continuity of the function $f(x,y)$ at the point $(0,0)$, where

$$f(x,y) = \frac{x+y}{x^2+y^2}, \quad \text{when } (x,y) \neq (0,0)$$
$$= 0, \quad \text{when } (x,y) = (0,0).$$

(b) If $u = \varphi(x+ct) + \psi(x-ct)$, then show that $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ where φ and ψ are two differentiable functions and c is a constant.

(c) Show that the set $S = \{(x,y) : x^2 + y^2 < 1\}$ is an open set which is not closed.

(d) If $z = x^3 - xy + y^3, x = rcos\theta, y = rsin\theta$, find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$.

(e) For what value of a, the vector $\vec{F} = xz\,\hat{\imath} + xyz\,\hat{\jmath} + ay^2\,\hat{k}$ is irrotational?

(f) Find the work done in moving a particle in the force field $\vec{F} = 2x^2\,\hat{\imath} + (xz+y)\,\hat{\jmath} + 3z\,\hat{k}$ along the curve $x = 2t,\ y = t,\ z = t^2$ from $t = 0$ to $t = 1$.

(g) Find the maximum value of the directional derivative of $\emptyset = xy^2 + 2yz - 3x^3z^2$ at $(1,-1,1)$.

(h) Find the circulation of $\vec{F}$ around the curve C, where $\vec{F} = (2x+y^2)\hat{\imath} + (3y-4x)\hat{\jmath}$ and C is the curve $y^2 = x$ from $(1,1)$ *to* $(0,0)$.

2. Answer *any four* of the following questions: **5 × 4=20**

(a) i) If $z = f(x,y)$ where $x = e^u \cos v,\ y = e^u \sin v$, show that

$$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}.$$

ii) Find the value of t so that the vector $\vec{F} = (x+3y)\hat{\imath} + (y-2z)\hat{\jmath} - (x+tz)\hat{k}$ is solenoidal. 3+2

(b) Evaluate

$$\iint_R [2a^2 - 2a(x+y) - (x^2+y^2)]\ dx\,dy$$

where R is the region bounded by the circle $x^2 + y^2 + 2a(x+y) = 2a^2$.

(c) Verify Green's theorem in the plane for

$$\oint_C [(xy + y^2)dx + x^2\, dy]\,,$$

where C is the closed curve (boundary) of the region bounded by $y = x$ and $y = x^2$, taken counterclockwise.

(d) If $f(x, y) = xy$ when $|x| \geq |y|$

$= -xy$ when $|x| < |y|$.

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

(e) Evaluate the surface integral $\iint \vec{F}.\hat{n}dS$ where $\vec{F} = zxy\hat{\imath} + yz\hat{\jmath} + x\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ in the second octant.

(f) Prove that a vector field is conservative if and only if it is irrotational.

3. Answer *any one* of the following questions: **10 × 1=10**

a) i) State Gauss' divergence theorem and use it to evaluate

$$\iint_S \vec{A} \cdot \hat{n}\ dS$$

where $\overrightarrow{A} = x^3\,\hat{\imath} + x^2\,y\,\hat{\jmath} + x^2z\,\hat{k}$ and S is the closed surface bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 2$. 1+5

ii) If $\vec{A}$ is a differentiable vector function and φ is a differentiable scalar function, then show that $\vec{\nabla} \times (\varphi\,\vec{A}) = (\vec{\nabla}\varphi) \times \vec{A} + \varphi\,(\vec{\nabla} \times \vec{A})$. 4

b) (i) If $\vec{F}$ is a continuous vector function defined on a smooth surface S then prove that $\iint_S \vec{F}.\hat{n}dS = \iint_R \vec{F}.\hat{n}\frac{dx\,dy}{|\hat{n}.\hat{k}|}$, provided $\hat{n}.\hat{k} \neq 0$ and R is the orthogonal projection of S on the xy plane. 6

(ii) Find the maximum or minimum value of $f(x, y, z) = x^m y^n z^p$ subject to the condition $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$ by using the method of Lagranges multiplier. 4

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